

Physics 5BL Lab 07

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Abstract

In this experiment, we investigated Hooke's Law and damped harmonic motion using an iOLab device attached to a spring in three configurations: horizontal (0°), vertical (90°), and on a 60° incline. First, the mass of a weight set was measured using force data from the iOLab, yielding a value within 5% of the nominal mass. The measured mass was 0.241 kg and the accepted value is 0.25 kg, so a numerical difference of 0.009 kg which only results in 3.6% difference in nominal value. For the horizontal configuration, a linear least-squares fit of force versus displacement produced a spring constant of $k = 10.90 \pm 0.06 \text{ N/m}$ consistent with the specified range of the manufacturer. In the vertical configuration, acceleration data were compared to overdamped, critically damped, and under damped models, with the under damped model providing the best agreement. On the 60° incline, both simple harmonic and friction-inclusive damping models were tested; the constant friction model best described the observed amplitude decay. Overall, the experiment confirmed the validity of Hooke's Law and demonstrated a discrepancy between ideal and damped oscillatory behavior in real systems as opposed to theoretical models.

0.1 Roles and Contributions

Both group members worked on part 0 and part 3. Alex primarily worked on descriptions for part 1 and Colton on part 2. Analysis for each part was done jointly. Alex worked on the conclusion, discussion, and theory. Colton worked on the abstract, discussion, and experimental setups.

1 Introduction and Theory

1.1 Fundamentals

Systems that involve oscillatory motion appear throughout physics and the world at large, from simple springs and pendulums to vastly complex structures of molecules, atoms, or the functionality of gluons inside protons and neutrons, oscillations appear in a wide variety of places. A fundamental example of this is a mass and spring system with no friction, which would align with Hook's law, and there is also systems with friction often called "damped" oscillation.

1.2 Hooke's Law

The classical Hooke's Law states that $F = -kx$, with F being the force induced by a, hookian, spring, k the spring constant, and x is the distance of one end of the spring to its equilibrium position.

1.3 Simple Harmonic Motion

From Newton's second law, $F = ma$, where $a = \frac{d^2x}{dt^2}$, where F is a general force, m is mass, and a is acceleration or $\frac{d^2x}{dt^2}$. Solving for $\frac{d^2x}{dt^2} = \frac{-kx}{m}$ results in $x(t) = A \cos(\omega t + \phi)$ where A is the amplitude of the oscillation, ω the frequency, t time, and ϕ is the initial phase of the oscillation. This equation is also known as the simple harmonic motion equation.

1.4 Damped Oscillation Models

The Damped oscillation equation is similar to that of the simple harmonic equation but with one key distinction, it includes friction. So instead of having $F = -kx$, $F = -kx - bv$, where β is the damping constant b divided by the mass m , v is the velocity or $\frac{dx}{dt}$ and $\omega_0 = \sqrt{\frac{k}{m}}$. Rearranged, using $F=ma$, results in $a + \beta v + \omega_0^2 x = 0$, and solving for x yields three different equations:

$$x(t) = \begin{cases} Ae^{-\gamma t} \cos(\omega_d t + \phi), & \gamma < \omega_0 \\ (A + Bt)e^{-\gamma t}, & \gamma = \omega_0 \\ Ae^{-(\gamma + \sqrt{\gamma^2 - \omega_0^2})t} + Be^{-(\gamma - \sqrt{\gamma^2 - \omega_0^2})t}, & \gamma > \omega_0 \end{cases}$$

where $\gamma = \frac{b}{2m}$ and $\omega_0 = \sqrt{\frac{k}{m}}$.

In this report we primarily use β where $\beta = 2\gamma = \frac{b}{m}$

2 Experimental Method

2.1 Equipment

The equipment used was only that provided at our station during the lab. The following equipment was used:

- Flat rectangular wooden board (with secure metal screw)
- iOLab data acquisition device
- Spring
- Metal dowel

- Retort/clamp stand
- Weight set
- Protractor

2.2 Measurement Overview and Procedure

For part 0, in which we measured the mass of the weight set, our procedure followed the instructions given on the lab sheet. The weight set was hung from the iOLab hook which pointed downwards and we recorded the force and acceleration on the iOLab with the weight set hung on. While still recording, the weight set was taken off of the iOLab which was then flipped 180° , held still at that position with the hook pointing upwards, and then rotated back to the original downward facing position.

For part 1 our goal was to measure the spring constant of the provided spring. We placed the iOLab, with wheels down, on the horizontal wooden board. We attached a spring to the hook on the iOLab and then to the secure metal screw attached to the wooden plank at a fixed length. The iOLab recorded the force on the hook, the position (by the wheels), and acceleration (using the accelerometer) as the iOLab was rolled by hand gently back and forth between two fixed points 10cm in distance from one another. Three periods of this motion were recorded per trial and three trials were recorded and data converted to a csv file.

For part 2, the goal was to record data at a 90 degree angle this time, so vertical instead of horizontal. A metal dowel was clamped by a screw to the retort/clamp stand on the lab bench. From the dowel, the spring was hung and from the spring the iOLab by hook. The iOLab was held at rest with the spring attached and not stretched initially. Then recording began and the force was re-zeroed at the rest position after which the iOLab was released from rest and allowed to oscillate for about 30 seconds. This process was repeated twice more to obtain three trials and compiled into a csv file for analysis.

For part 3, we used the iOLab position, force, and acceleration detectors just as we did in part 1 but this time on a 60 degree incline with the wooden board. We attached the spring to a hook at the top of the wooden board and let the iOLab oscillate up and down the board diagonally attached to the spring. We allowed about 10 oscillations, which gave enough time for the iOLab to lose most of its energy and slow down to almost rest after this many periods. We took data for 3 trials of this roughly 10 period oscillation.

3 Part 0: Mass Measurement with iOLab

3.1 Force Measurement and Analysis

The period of time in the recorded data in which the weight set was hung from the spring at rest was roughly 5 to 6.5 seconds. A quiet and relatively flat region

of the force data around 5.9-6.4 seconds was used for the average force by the weight set. This average force was $2.32 \text{ N} \pm 0.03 \text{ N}$. The nominal mass of the weight set is 250 g which, by Newton's 2nd Law, $F = ma$, should give a force of about 2.45 N.

3.2 Discussion

The results would not be accepted as the correct value however they are reasonably accurate considering the setup of the experiment. Due to the iOLab being held by hand, it is impossible to keep it completely at rest due to human error meaning that the mass will hang at some angle from the vertical. This offset will reduce the measured force. Additionally, the force could be calculated as being smaller from that of the nominal force due to the possibility of the iOLab moving vertically slightly in the quiet period which our data does suggest as the force slightly increases over the time range we selected. This would also lower the force. Between these errors, a discrepancy of about 5% can be expected which is what we saw.

The accelerometer measurement is not 0 when the device is being held still due to the gravitational field of the Earth which has acceleration $g = 9.8 \text{ m/s}^2$. When the device is being supported and is not in free fall but rather hanging at "rest" by a hand, the measured acceleration will therefore be around 9.8. The accelerometer reads the proper acceleration (the normal/support force per unit mass) and not the net acceleration in an inertial frame of reference.

4 Part 1: Hooke's Law at 0°

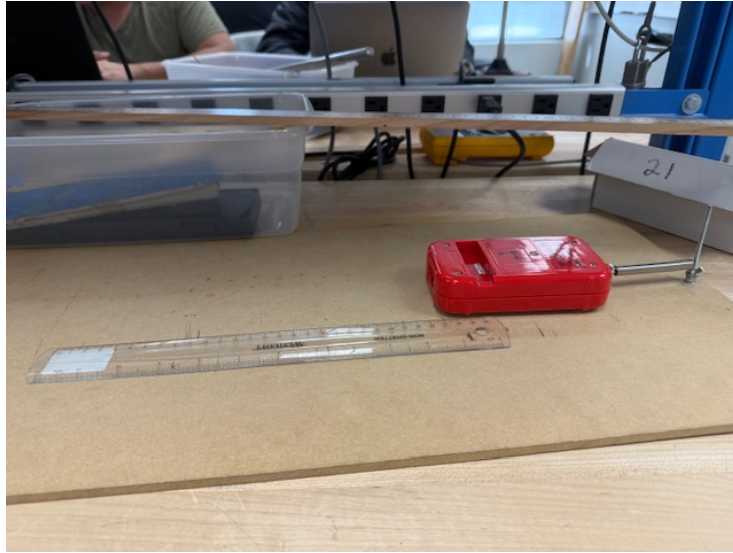


Figure 1: The 0° setup with the iOLab wheels down on top of a wooden board. A spring attached to the iOLab is connected to a stationary metal bolt attached to the wooden board. The ruler was used to measure the distance the iOLab was oscillated between.

4.1 Force vs. Displacement Data

The data collected ranged from three different trials each with 3 back and forth motions of the iOLab. Data for the force and displacement, each with respect to time, were collected and the plots for the best trial is shown below. A clear sinusoidal-like pattern can be seen in both plots in figure 1 as expected from our methodology. This allows us to analyze a certain range of time in the data that we can clearly select based on loading vs unloading patterns in the data.

4.2 Linear Fit to Hooke's Law

In order to calculate the spring constant k from Hooke's Law, a linear least squares fit was applied to a force vs. position plot over a smooth time range. The start time was 4.4 s and the end time was 4.8 s for the time period used to analyze the force and position relationship. Figure 2, shown below, is a plot of the force vs. position over the selected time period with a line of best fit using the linear least-squares fit method.

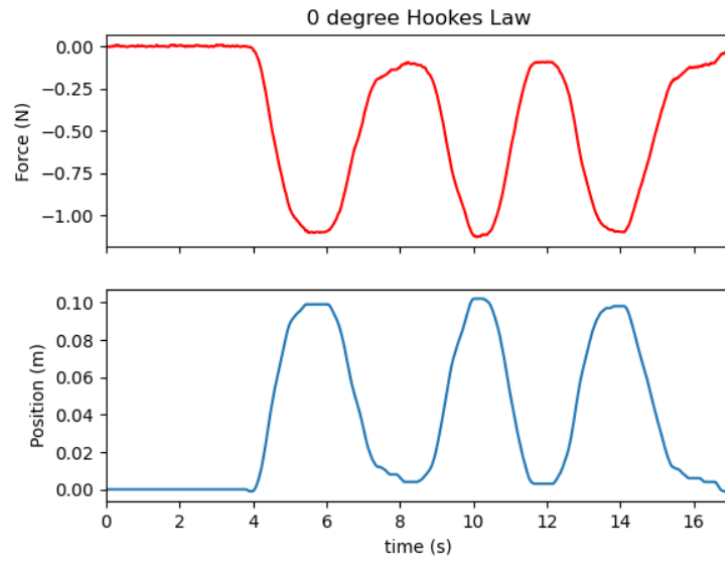


Figure 2: Force (N) and Position (m) vs. Time (s)

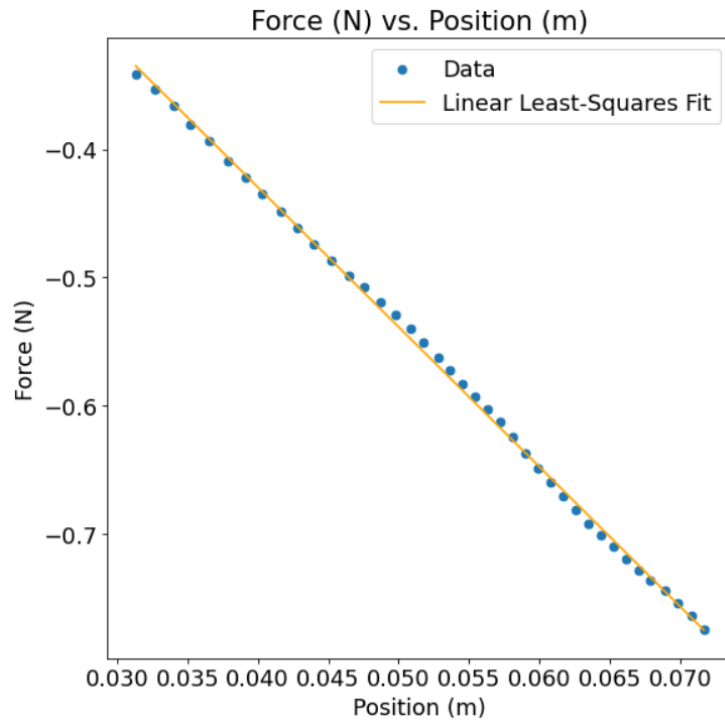


Figure 3: Force (N) vs. Position (m) scatter plot and a linear least-squares fit

From Hooke's Law, $F = -kx$, we find that $k = -F/x$. We can use the slope of the linear least squares fit line which is $m = F/x$ and multiply it by (-1) to get the value for k . The fit line has a slope $m = -10.90 \pm 0.06$ giving $k = 10.90 \pm 0.06$ N/m. For the y-intercept, we get $c = 0.007 \pm 0.003$.

Although Hooke's law would imply that there would be no y-intercept as when $x=0$ then $F=0$, however in our data there is a non-zero y-intercept. This is likely caused by imperfect calibration of sensors, flaws with the zeroing of data, errors with the spring to make it non-hookean, etc. The resulting intercept was 0.007 ± 0.003 Nm, suggesting that these had minor effects.

4.3 Residual Analysis

We plotted the residuals for the force vs position, as seen in figure 3. It is clear from the residuals that the model for pure harmonic oscillation with a Hookean spring does not accurately fit the data we collected. This is obvious from the clear pattern in the residuals plot, whereas should the model fit well the residuals are randomly scattered. There are many reasons why the model may not have worked to predict what data we would have collected. The spring was not oscillating freely, rather it underwent oscillation-like motion at the will of our hand which we used to pull the iOLab and stretch and compress the spring. The residuals have a clear pattern which is most likely from the manual motion and type of friction that the iOLab experienced which was in its wheels and not over the whole object itself. The rolling inertia of the object caused additional forces when accelerating and decelerating, because some of the applied force went into rotation motion rather than the purely translational motion we predict with Hooke's law. This is a systematic variation in the data that the manual hand motion could amplify contributing to a pattern in the residuals.

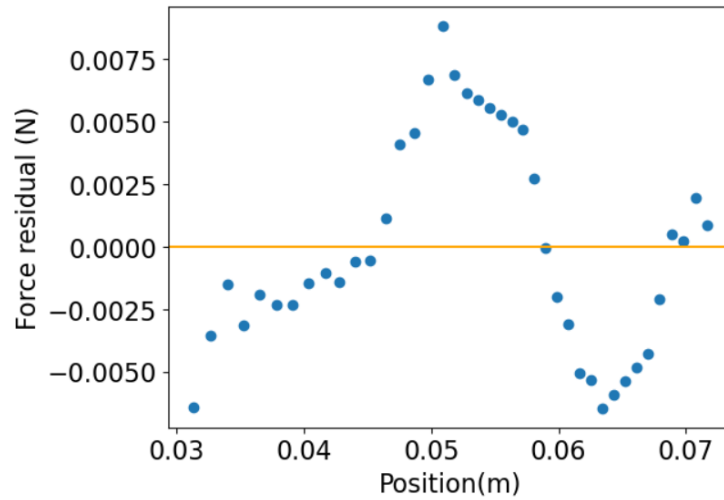


Figure 4: Force Residuals (N) vs. Position (m). The plot determines whether or not the Hookean spring model accurately fits the data collected.

4.4 Acceptance Test

The manufacturer specifies $k = 12 \pm 10\%$, corresponding to an acceptable range of 10.8–13.2 N/m. Our measured value, $k = 10.90 \pm 0.06$ N/m, falls within this interval and therefore satisfies the manufacturer acceptance. However, the difference between our measured value and 12 N/m is much larger than our statistical uncertainty, indicating that systematic effects likely cause most of the discrepancy. Furthermore, the residuals from the linear least squares fit exhibit a non-random pattern rather than a uniform distribution about zero, suggesting slight deviations from ideal Hookean behavior or limitations of the linear model over the selected displacement range.

5 Part 2: Vertical Harmonic Oscillation at 90°



Figure 5: Picture of the set up for the 90° in lab. The iOLab was suspended by a spring attached to the metal dowel and was dropped from rest allowing it to oscillate up and down freely.

5.1 Notes

In order to perform the calculations for this section of the report certain values needed to be known, such as the spring constant k . As $F = -kx$ we used a ruler to find the displaced distance and used the force readings from the iOLab to estimate the value of k . However, this could have been found using the fact that, as is later found, the situation is an under damped harmonic oscillator which follows an equation stating that the acceleration in the y direction is proportional to k in some way (such as trig functions and other functions).

5.2 Force vs Acceleration

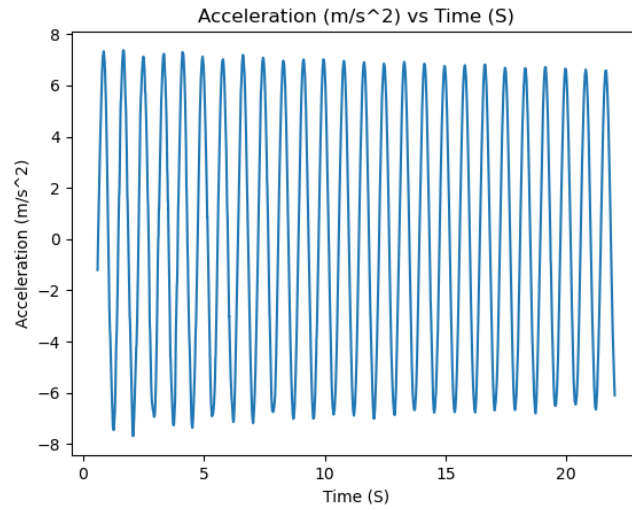


Figure 6: Acceleration ($\frac{m}{s^2}$) data plotted with respect to time (s) for 90° situation

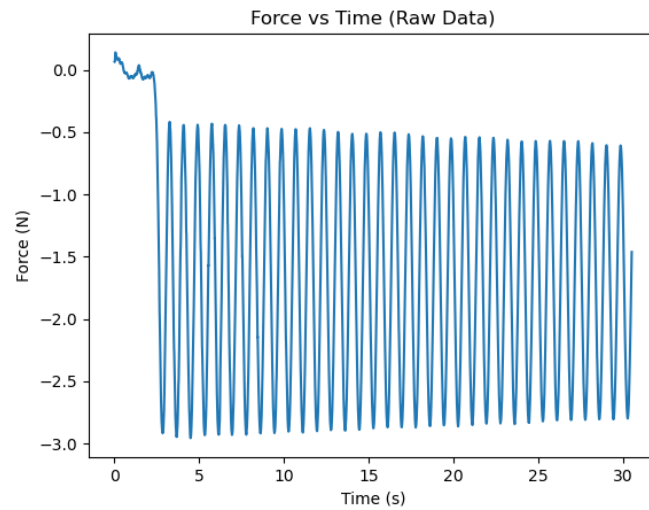


Figure 7: Force data plotted with respect to time (not cut off data)

Both graphs are wave-like in nature and have slowly decreasing amplitudes over time. However, they differ in phase as the accelerometer only reads its actual

position, while as the force sensor reads based on how much the hook moves, and as the hook is separate from the iOLab it lags differently compared to the rest of the iOLab due to inertia. Additionally, they likely have offset timing due to internal inconsistencies within the iOLab.

5.3 Over Damped

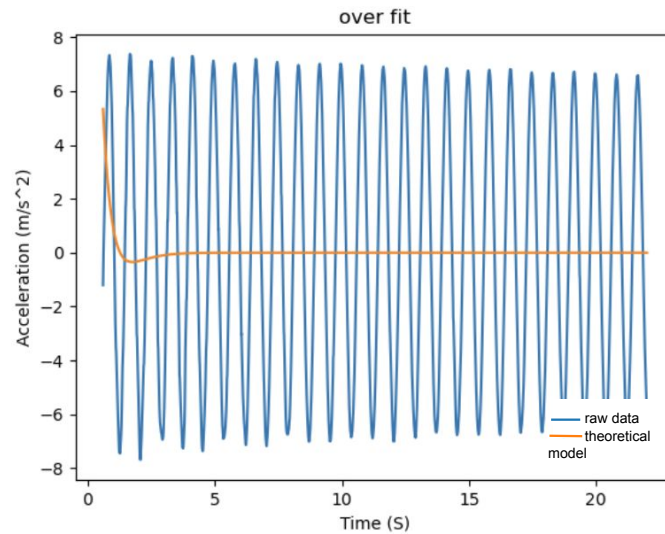


Figure 8: Over damped theoretical model (orange) compared to actual data (blue)

The theoretical model for over damped oscillation shown in figure 6 in orange does not remotely match the observed data shown in blue. The observed data represents clear oscillations that slightly decrease in magnitude only over at least 20 periods while the over damped model predicts motion from only one peak and one trough to culminate in enough work done by friction to slow the oscillator to rest. Therefore, our observed data is highly unlikely to fall in the over damped regime for oscillatory motion.

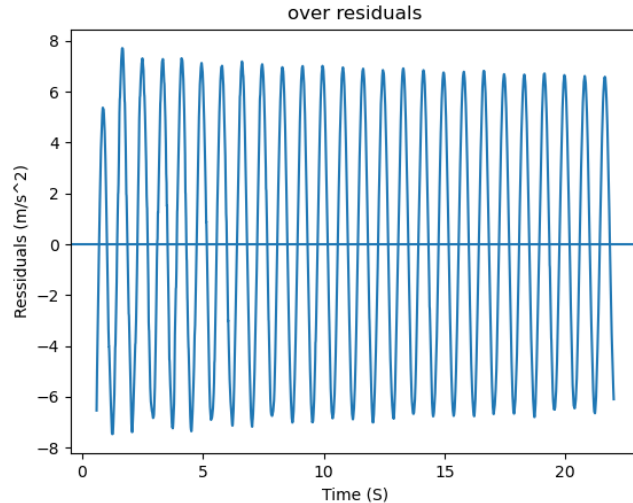


Figure 9: Residuals of prior graph

While it was clear to see from directly comparing the acceleration vs. time plots for both the over damped regime and our observed data, the residuals further demonstrate that the model is undoubtedly inaccurate as the residuals, except for at the first period, match that of the regular acceleration vs. time graph of the observed data due to the over damped model zeroing out after a full period.

5.4 Under Damped

In Figure 8, the theoretical model (orange) for the under damped oscillation regime can be seen layered over a graph of our observed data (blue) which we matched through a least-squares fitting method for each plot of acceleration vs. time. The theoretical model matches that of the observed data with high accuracy to the extent that it covers most of the observed data graph (in blue). The under-damped model best represent the data thus far however plotting of the residuals is necessary to truly test the data.

The residuals plot for our observed data shown in figure 8 is shown in figure 9. The residuals plot is not randomized and scattered uniformly above and below the x-axis as we would want for an ideal and exact model for observed data. Despite this inconsistency, we can learn a lot from this residuals plot. The inconsistencies are likely from slight inaccuracies in our model for the observed data that result in difference phases for the model vs the observed data. This is because if our parameters that were fit for the theoretical model we slightly inaccurate, which they are bound to be with limited fitting capabilities, then the phase of the fit function over each period is necessarily going to be off.

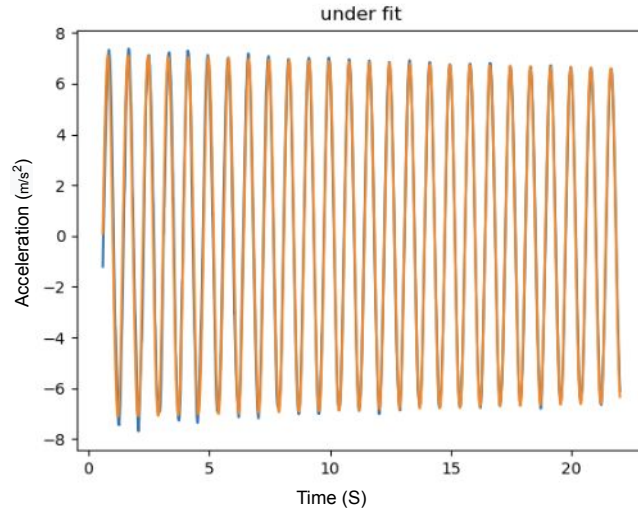


Figure 10: This graph represents the raw data compared to the under damped theoretical model

In our residuals plot, we see what we would expect from this specific type of inconsistency where the residuals plot itself has oscillatory characteristics. This shows that throughout each period, the theoretical model either briefly lags or rushes compared to the observed data which means that these parameters we fit for could just be off. This does not necessarily mean that the model itself is wrong though, just that using a better fitting algorithm could improve results in the future.

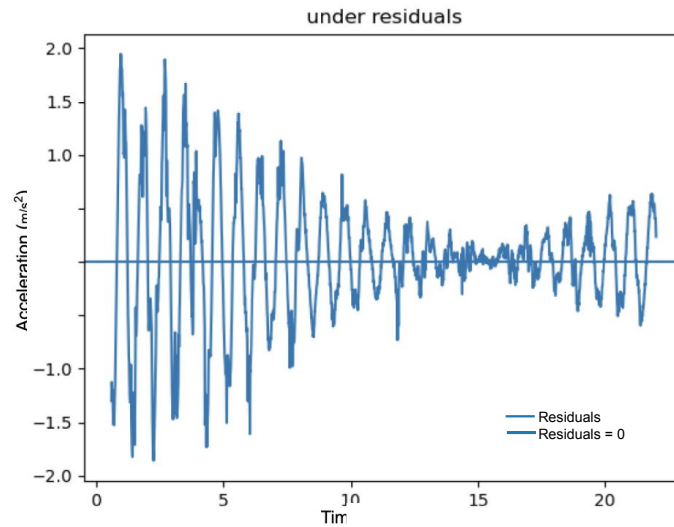


Figure 11: Residuals of prior graph

5.5 Critically Damped

Figure 12 shows (in blue) the observed data for acceleration in meters per second squares with respect to time in seconds and (in orange) the theoretical expectation for the critically damped regime of oscillation. The model for critical damping bears no resemblance to the observed data and not just in phase or amplitude, which can be changed and treated as fitting parameters, but by the complete shape of the model. The residuals in figure 13 agree with this demonstrating an accurate resemblance between the residuals and the observed data plotted for acceleration vs time.

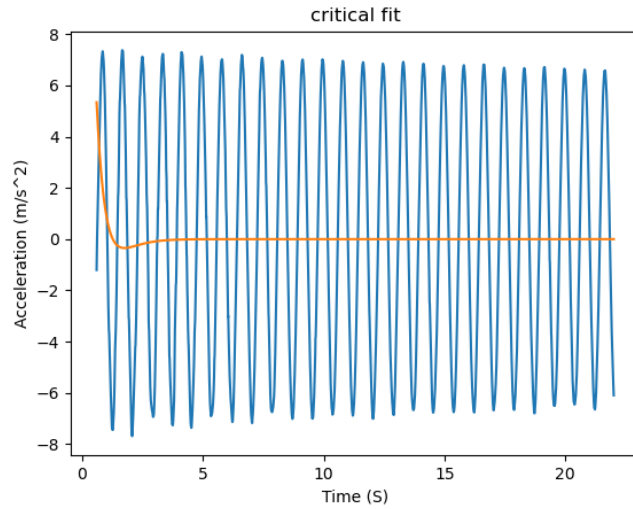


Figure 12: This graph represents the fit of the critically damped oscillation (orange) compared to the raw data (blue)

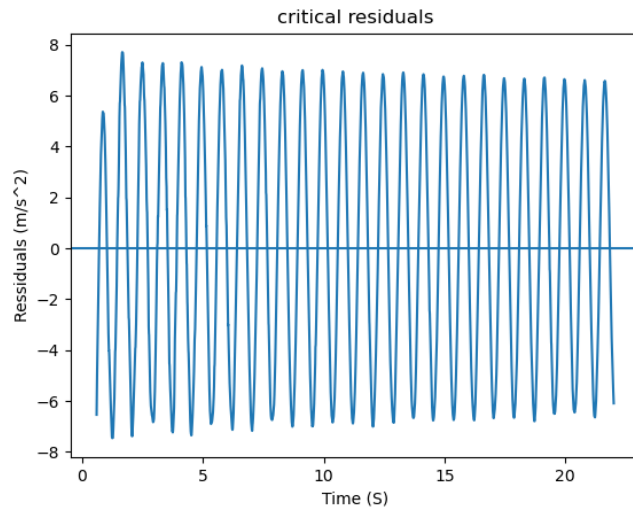


Figure 13: Residual of the critically damped 90° system

5.6 Data

Data for the fitted model for simple harmonic oscillation:

$$\omega = 7.5382 \pm 0.0082 \frac{\text{rad}}{\text{s}}$$

$$T = 0.83351 \pm 0.00091 \text{ s}$$

$$k = 13.581 \pm 0.03 \frac{\text{N}}{\text{m}}$$

Data for the fitted model for damped harmonic oscillation:

$$\omega = 7.5469 \pm 0.0051 \frac{\text{rad}}{\text{s}}$$

$$\beta = 0.19197 \pm 0.0052 \frac{1}{\text{s}}$$

$$T = 0.83255 \pm 0.00056 \text{ s}$$

$$k = 13.612 \pm 0.018 \frac{\text{N}}{\text{m}}$$

5.7 Conclusion

Based on the accurate fit seen in figure 10 for the theoretical model for under damped oscillations on the observed data, we can conclude that the model supports our data and that the experiment involved under damped oscillations of the iOLab. That being said, the residuals do not completely agree with the former statement as they are not uniformly distributed about the x axis. However, it is expected to see some patten in the residuals for this type of motion for a fit that does match the shape of the observed data due to errors in accuracy from floating parameters. These errors are expected and likely resulted in the patterns we saw in the residuals plot despite the under-damped regime closely fitting our data overall.

6 Part 3: 60° Ramp Oscillation



Figure 14: Picture for the set up of the 60° in lab. The wooden board was propped up against the retort stand at an angle of 60 degrees. The same bolt attached to the wooden board as in part 1 was used to attach the spring and iOLab. The iOLab oscillated diagonally on this wooden board with wheels down allowing it to roll freely.

6.1 Phase Analysis of Position, Velocity, Acceleration, & Force

The damped wave graphs of each position, velocity, acceleration, and force all align well with each other for each period. Choosing the first full smooth period of time to analyze oscillations, the period of the oscillations are equal for each wave (p, v, a, F). That being said, due to lack of measurement accuracy and errors in equipment, there is a very minimal phase shift between these each of the respective values' peaks. The greatest difference in these phase shifts is 0.02 seconds, an amount which can be accounted for by the time at which the iOLab registers information. For example, when the position is at one of the local extrema, the velocity may not be counted as exactly 0 at this time due to faults in the iOLab's measurement accuracy. This being said, the periods' lengths to match relative to each other for their length. Now we can disregard the phase shift differences, for which the magnitudes are equal across each time period throughout the extent of the trials, implying systematic error from iOLab, and not any physical mean. Now, other than the minimal relative phase shifts, the

phases fit our predictions from 1-dimensional kinematics and Newton's laws. When the force exerted on the hook of the iOLab is at a peak or trough, the position and acceleration are also at their respective peaks or troughs for each period of time and the velocity is at 0 at these same times. This matches what we would predict from basic mechanics as a peak in position implies that we are at a turning. This also means that the velocity of the iOLab should be 0 because it has to change direction from positive to negative and vice versa. Additionally, the acceleration is maximized at these points due to maximum distance from the equilibrium position for the spring with the mass which also means the force is maximized. Therefore, the relative phases match prediction.

6.2 Simple Harmonic Fit

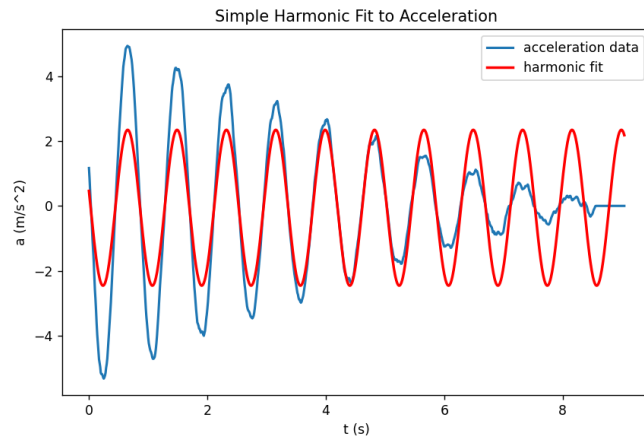


Figure 15: Acceleration (m/s^2) vs. time (s) for data and simple harmonic oscillation for 60° situation

This resulted in:

$$\omega = 7.5382 \pm 0.0082 \frac{rad}{s}$$

$$T = 0.83351 \pm 0.00091 s$$

$$k = 13.581 \pm 0.03 \frac{N}{m}$$

Compared to the k value found in the 0° this k value is about $2.9 \frac{N}{m}$ larger. However, only the 0° one is in the range while the 90° is not. Additionally, the methods for determining the value of k was different between each scenario. For 0° k, it was found simply using the equation $F = -kx$ and solving for k,

but for the 90° one, it was found using $\omega^2 = \frac{k}{m}$ with the measured value of m and the value of ω from the fitted curve.

6.3 Velocity-Dependent Damping Fit

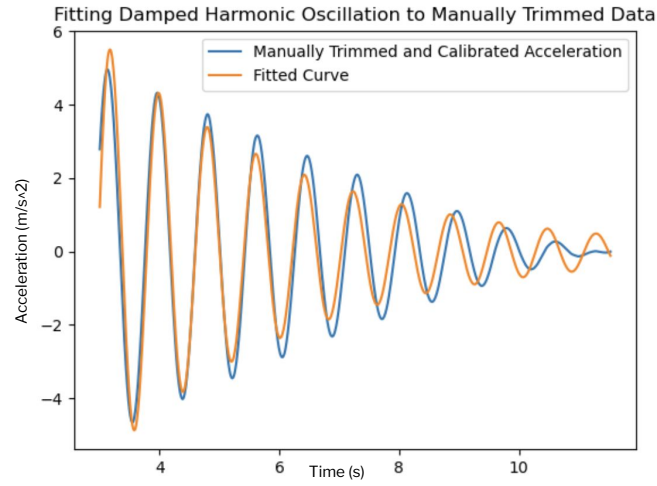


Figure 16: Fitted graph for the 60° Acceleration ($\frac{m}{s^2}$) vs Time (s) for velocity-dependent friction .data manually cut off as oscillation started after data began collecting

6.4 Constant Friction Analysis

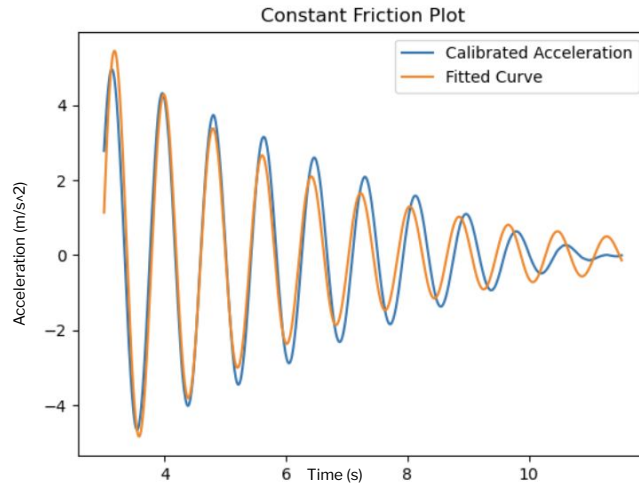


Figure 17: Fitted graph for the 60° Acceleration ($\frac{m}{s^2}$ vs Time (s) for constant friction.

t_n (s)	A_n
2.72	-2.196
3.55	-2.080
4.39	-1.978
5.23	-1.875
6.88	-1.679
7.72	-1.588
8.57	-1.504
9.40	-1.423

Table 1: Turning point amplitudes

From the turning point data we used the formula in the lab notes, $A_n - A_{n+1} = \frac{2f}{k}$ where A is the amplitude of the n-th turning point, to calculate the friction f.

We got values of:

$$\begin{aligned}\omega &= 7.5382 \pm 0.0051 \frac{\text{rad}}{\text{s}} \\ \beta &= 0.19197 \pm 0.0052 \text{ s}^{-1} \\ T &= 0.83255 \pm 0.00056 \text{ s}\end{aligned}$$

$$k = 13.612 \pm 0.018 \frac{N}{m}$$

6.5 Regime Determination

The most accurate representation of the data was the constant friction model. This is likely due to the fact that in the physical situation the iOLab remained in contact with a wooden board for the entire duration of the number of oscillations. However its not a perfect model as there is still a non-constant force of friction from the air, so a combination of the constant friction and the damped harmonic oscillation would be the most accurate.

7 Discussion

Part 0 of this lab was centered around understanding the workings of the iOLab system and getting familiar with lab setups involving its data collection methods and tools. In the setup for the mass measurement of a weight set using the iOLab, we were able to determine within about 5 percent of the correct mass. Our uncertainty did not span the range that would also have accounted for the nominal mass of the weight and there were many reasons for this. The nominal value of the mass set assumed before could have been incorrect which past experiments demonstrated as a likely possibility, the unstable nature of holding the iOLab induces lots of error, and any sway side to side would decrease the vertical force on the iOLab reducing the mass calculated based on simple Newton's 2nd Law.

In the 0° horizontal configuration, the linear relationship between force and displacement verified Hooke's Law. The measured spring constant, $k = 10.90 \pm 0.06 \text{ N/m}$, fell within the manufacturer's specified tolerance range of $12 \pm 10\% \text{ N/m}$. Although the statistical uncertainty was small, the difference between our measured value and $12 \frac{N}{m}$ is much larger than the reported uncertainty, indicating that systematic effects dominate. The patterned residuals further support this conclusion. Likely sources of systematic error include rolling friction in the iOLab wheels, rotational inertia of the wheels (which diverts some translational energy into rotational motion), slight misalignment of motion along the intended axis, and manual forcing of the oscillations. These effects do not allow the assumption of a purely Hookean spring system that gives a restoring force proportional to the displacement from the equilibrium position and contributed to the structure in the residuals.

In the 90° vertical configuration, the system clearly showed oscillatory motion with gradually decreasing amplitude, ruling out both overdamped and critically damped regimes. The under damped model provided the best model or agreement with the observed acceleration data, as confirmed by visual agreement and residual analysis. Additionally from the data provided it showed that it was under damped. Each regime can be determined by what the value of $\frac{\beta}{2\omega_0}$ is equal to. In this case $\beta \approx 0.19$ and $\omega_0 \approx 7.5$, so $\frac{\beta}{2\omega_0} \approx 0.013$ which would correspond to an under damped oscillation. The oscillatory structure remaining in

the residuals suggests small inaccuracies in fitted parameters, particularly phase and damping constants. Because damped oscillatory motion is highly sensitive to parameter values, even slight deviations in angular frequency or damping coefficient produce systematic phase drift over multiple periods. Furthermore, other values were found such as the spring constant $k = 13.581 \pm 0.03 \frac{N}{m}$, and the period $T = 0.83351 \pm 0.00091$ s. Additional sources of error include lateral (side to side types of) oscillations, small rotations of the iOLab during motion, and internal sensor timing offsets between force and acceleration measurements.

For the 60° incline configuration, damping effects were more pronounced due to continuous contact between the iOLab and the wooden board. While a simple harmonic model produced reasonable estimates of angular frequency and spring constant ($k \approx 13.6 \text{ N/m}$), the amplitude decay was better described by a constant friction model. The measured damping parameter $\beta = 0.19197 \pm 0.0052 \text{ s}^{-1}$ indicates significant energy loss per cycle. The preference for the constant friction model is physically reasonable because rolling contact introduces approximately constant resistive forces, while air resistance likely contributes a smaller velocity-dependent component. The discrepancy between the spring constant measured in the 0° and 60° configurations (approximately 2.7–2.9 N/m difference) further indicates that different measurement techniques and frictional forces influence the inferred value of k . Especially, determining k from $\omega^2 = k/m$ assumes negligible damping, an approximation that is not perfectly satisfied in the incline setup.

Across the three setups, most of the uncertainty in our results was clearly systematic rather than statistical, even though the numerical uncertainties from the fits were small. In reality, the motion was affected by rolling friction in the wheels, sliding contact with the board, air resistance, slight off-axis oscillations, and even small rotations of the iOLab that we couldn't completely prevent. On top of that, every release was done by hand, so there was always some tiny misalignment or unintended push at the start. Because of this, the quoted statistical errors from the fits are probably underestimating the true uncertainty, since the dominant effects come from these consistent physical imperfections rather than random noise in the data.

Overall, the experiment shows that while Hooke's Law and simple harmonic motion do a very good job describing the basic behavior of a spring-mass system, they don't capture everything once real world effects are included. The horizontal setup behaved closest to the ideal Hookean case, the vertical configuration clearly showed under damped oscillations, and the incline made friction much more obvious, to the point where a constant-friction model described the decay better than a purely velocity-dependent one (which makes sense when we use wheels and not direction iOLab contact with the wooden board). In other words, the theory works, but only to the extent that we account for the physical details of the system. Classical mechanics still predicts the overall behavior correctly, but getting precise agreement requires acknowledging the non-ideal effects that are always present in these kinds of actual experiments.

8 Conclusion

The purpose of this experiment was to obtain values of multiple variables such as the spring constant k or the damping constant b ($\beta = \frac{b}{m}$) and also to find which theoretical models are appropriate for the three scenarios. For the 0° situation, the most appropriate model was simply using Hooke's law, for the 90° ; the under damped, damped harmonic oscillator model was best, and lastly for the 60° the model the most closely aligned with the data was the damped harmonic oscillation with constant friction, however, that is not to say they were the perfect models.

While the models fit well with the data, they were not perfect, and this was likely due to some form of error. The most pronounced error came from the oscillations not being perfectly aligned with one directional coordinate. For example in the 90° experiment the iOLab would also oscillate in other directions, and sometimes even rotate. While we tried our best to avoid this it was nearly impossible to ensure that there was no unwanted movement. Additionally there are some systematic errors, like in the 3rd experiment where the model only accounted for friction induced by the contact between the iOLab and the wooden slab provided in the lab, it didn't account for the air resistance like the damped harmonic oscillation did. As a result the model wasn't perfect, but realistically the air resistance should be significantly smaller than the other friction.